Exercises for the lecture course Algebraic Topology I – Sheet 9

University of Bonn, winter term 24/25

Aufgabe 33. Compute for $n \geq 2$ and $k \geq 1$:

(a) $\pi_1(S^{n-1} \times \mathrm{SO}(n) \times \mathbb{RP}^n \times \mathbb{CP}^n)$;

(b) $\pi_k(T^n \times \mathbb{RP}^\infty \times \mathbb{CP}^\infty)$;

(c) $\pi_2(S^n \vee \mathbb{CP}^n)$.

Aufgabe 34. Prove or disprove that the obvious map $\pi_3(D^2, S^1) \to \pi_3(D^2/S^1)$ is surjective.

Aufgabe 35. Consider $m, n \in \mathbb{Z}^{\geq -1}$. Let X and Y be spaces such that X is m connected and Y is n-connected, where (-1)-connected means that there is no condition. The join $X * Y$ of X and Y is defined by the pushout

$$\begin{array}{ccc} X \times Y & \longrightarrow & X \times \mathrm{cone}(Y) \\ \downarrow & & \downarrow \\ \mathrm{cone}(X) \times Y & \longrightarrow & X * Y. \end{array}$$

Prove that the join $X * Y$ is $(m + n + 2)$-connected.

Aufgabe 36. Prove or disprove:

(a) For every simply connected topological group G we have $\pi_1(\Omega BG) = \{1\}$;

(b) If G is a topological group, then $\pi_1(G)$ is abelian;

(c) If G is a compact connected Lie group and the universal principal G-bundle $p\colon EG \to BG$ has a section $s\colon BG \to EG$, then G is the trivial group.

[0]Hand-in Monday 09.12.