Exercises for the lecture course Algebraic Topology I – Sheet 8

University of Bonn, winter term 24/25

Aufgabe 29. Let G be a path connected topological group and $p\colon E \to B$ be a principal G-bundle. Prove or disprove that the fiber transport associated to p regarded as a fibration is trivial.

Aufgabe 30. Let $H\colon \mathbb{Z} \to \pi_3(S^2)$ be the isomorphism sending $1 \in \mathbb{Z}$ to the class $[p]$ of the Hopf fibration $p\colon S^3 \to S^2$. Let $f\colon S^3 \to S^3$ and $g\colon S^2 \to S^2$ be maps. Prove: $H(\deg(f)) = [p \circ f]$ and $H(\deg(g)^2) = [g \circ p]$.

Aufgabe 31. Decide for which $d \in \mathbb{Z}^{\geq 1}$ any principal G-bundle over any d-dimensional CW-complex is trivial, where G is $\mathbb{Z}$ with the discrete topology, S^1, or S^3 with the multiplication coming from the embedding $S^3 \subseteq \mathbb{H}$ into the field of quaternions.

Aufgabe 32. Let $p\colon E \to B$ be a fibration over a path connected space B. Let $F = p^{-1}(b)$ for some $b \in B$. Recall that a space X is called aspherical if it is path connected and $\pi_n(X, x)$ vanishes for all base points $x \in X$ and $n \geq 2$. Prove or disprove:

(a) If F and B are aspherical, then E is aspherical;

(b) If F and E are aspherical, then B is aspherical;

(c) If E and B are aspherical, then F is aspherical.

[0]Hand-in Monday 02.12.