Exercises for the lecture course Algebraic Topology I – Sheet 7

University of Bonn, winter term 24/25

Aufgabe 25. Consider the subspace $X = I \times \{0\} \cup \{0\} \times I$ of $\mathbb{R}^2$. Let $f\colon X \to I$ be the map sending (x, y) to x.

Prove or disprove that f is a fibration.

Aufgabe 26. Let F be a finite set equipped with the discrete topology. Put $X = \prod_{n \in \mathbb{Z}} F$ with respect to the classical product topology. Let X_d be the set X equipped with the discrete topology and let $p\colon X_d \to X$ be the map given by the identity. Prove:

(a) X is a compact Hausdorff space;

(b) X is a compactly generated space;

(c) X is totally disconnected, i.e., each of its components contains only one point;

(d) Each path components of X contains only one point;

(e) p is continuous and bijective;

(f) p is a not homeomorphism;

(g) p is a fibration.

Aufgabe 27. Prove or disprove:

(a) The composite of two fibrations is again a fibration;

(b) The product of two fibrations is again a fibration;

(c) A fibration with non-empty domain and locally contractible codomain is injective if and only if it is a homeomorphism.

Aufgabe 28. Let $p\colon E \to S^1$ be a fibration. Let $F_s = p^{-1}(s)$ be the fiber of s. Let the homotopy equivalence $f\colon F_s \to F_s$ be a representative of the fiber transport associated to a generator of $\pi_1(S^1, s)$.

Prove that E is homotopy equivalent to the mapping torus T_f of f.

---

[0]Hand-in Monday 25.11.