Exercises for the lecture course Algebraic Topology I – Sheet 6

University of Bonn, winter term 24/25

Aufgabe 21. Consider the pushout

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ {\scriptstyle i}\downarrow & & \downarrow{\scriptstyle \bar{i}} \\ X & \xrightarrow[\bar{f}]{} & Y. \end{array}$$

Suppose that i is the inclusion of a strong neighborhood deformation retraction (X, A).

Prove or disprove that $\bar{i}$ is the inclusion of a strong neighborhood deformation retraction (Y, B).

Aufgabe 22. Let $i\colon A \to X$ be a cofibration. Let $f\colon (X, A) \to (Y, B)$ be a map which is as a map of pairs homotopic to a map $g\colon (X, A) \to (Y, B)$ satisfying $g(X) \subseteq B$.

Prove or disprove that f is homotopic relative A to a map $g\colon (X, A) \to (Y, B)$ satisfying $g(X) \subseteq B$.

Aufgabe 23. Let X and Y be well-pointed spaces. Prove or disprove that their smash product $X \wedge Y$ is well-pointed.

Aufgabe 24. Consider the commutative diagram

$$\begin{array}{ccccccc} X_0 & \xrightarrow{i_0} & X_1 & \xrightarrow{i_1} & X_2 & \xrightarrow{i_2} & \cdots \\ \downarrow{\scriptstyle f_0} & & \downarrow{\scriptstyle f_1} & & \downarrow{\scriptstyle f_2} & & \\ Y_0 & \xrightarrow{j_0} & Y_1 & \xrightarrow{j_1} & Y_2 & \xrightarrow{j_2} & \cdots \end{array}$$

Suppose that each horizontal arrow is a cofibration and each vertical arrow is a homotopy equivalence.

Prove or disprove that the induced map

$$\operatorname{colim}_{n\to\infty} f_n\colon \operatorname{colim}_{n\to\infty} X_n \to \operatorname{colim}_{n\to\infty} Y_n$$

is a homotopy equivalence.

[0]Hand-in Monday 18.11.