Exercises for the lecture course Algebraic Topology I – Sheet 4

University of Bonn, winter term 24/25

Aufgabe 13. Let X be a topological space which is dominated by a CW-complex Y, i.e., there exists a CW-complex Y with maps $i\colon X \to Y$ and $r\colon Y \to X$ with $r \circ i \simeq \mathrm{id}_X$.

Prove or disprove that X has the homotopy type of a CW-complex.

Aufgabe 14. Prove or disprove that a compact metric space Y has a CW-approximation $f\colon X \to Y$ with compact X.

Aufgabe 15. Let X be a m-connected and Y be a n-connected CW-complex coming with base points. Prove or disprove that $X \wedge Y$ is $(m+n+1)$-connected.

Aufgabe 16. Let X be the quotient space obtained from $S^1 \subseteq \mathbb{R}^2$ by identifying any two points in the open subset $\{(x,y) \in S^1 \mid y > 0\}$ and by identifying any two points in the open subset $\{(x,y) \in S^1 \mid y < 0\}$. Let $p\colon S^1 \to X$ be the projection. Then the set X has four points, namely, the images of $(0,1)$, $(0,-1)$, $(1,0)$, and $(0,-1)$ under p.

(a) Describe the open subsets of X and show that X is not a Hausdorff space, is path connected, and is pre-compact, i.e., every open covering has a finite subcovering;

(b) Prove or disprove that X has a universal covering $p\colon \widetilde{X} \to X$;

(c) Prove or disprove that $p\colon S^1 \to X$ is a CW-approximation;

(d) Prove or disprove that S^1 and X are homotopy equivalent.

[0] Hand-in Monday 04.11.