Exercises for the lecture course Algebraic Topology I – Sheet 3

University of Bonn, winter term 24/25

Aufgabe 9. Prove or disprove for $n \geq 0$ that an n-connected CW-complex of dimension n is contractible.

Aufgabe 10. Let X be a CW-complex such that for every natural number m there is a natural number n with $m \leq n$ such that the inclusion $X_m \to X_n$ is nullhomotopic. Prove or disprove that X is contractible.

Aufgabe 11. Let G be a finitely generated abelian group G and $n \in \mathbb{Z}^{\geq 1}$. Construct a compact $(n+1)$-dimensional path connected CW-complex X with $\pi_n(X) \cong G$.

Aufgabe 12. Let N be a path connected oriented closed smooth n-manifold which has the following property: For any path connected oriented closed smooth n-manifold M the degree defines a bijection $\deg\colon [M, N] \to \mathbb{Z}$.

Prove or disprove that N is oriented homotopy equivalent to S^n.

[0]Hand-in Monday 28.10.