Exercises for the lecture course Algebraic Topology I – Sheet 2

University of Bonn, winter term 24/25

Aufgabe 5. Let W be the *Warsaw circle*, i.e., the union of subsets of $\mathbb{R}^2$ given by the union of $\{(x, \sin(2\pi/x)) \mid x \in (0,1]\}$, $\{(1,y) \mid y \in [-2,0]\}$, $\{(x,-2) \mid x \in [0,1]\}$ and $\{(0,y) \mid y \in [-2,1]\}$.

Show that the projection $p\colon W \to \{\bullet\}$ is a weak homotopy equivalence but not a homotopy equivalence.

Aufgabe 6. Let $X \subseteq \mathbb{R}^{n+1}$ be the union $\bigcup_{k=0}^{\infty} Y_k$, where Y_k is the sphere around $(1/k, 0, 0, \ldots, 0)$ of radius $1/k$.

Prove or disprove that there is a surjective homomorphism $\pi_n(X,x) \to \prod_{i=0}^{\infty} \mathbb{Z}$ and hence $\pi_n(X,x)$ is uncountable for any base point $x \in X$.

Aufgabe 7. Compute the set of homotopy classes $[X,Y]$ of maps $X \to Y$ for the following cases:

(a) $X = Y = S^n$ for $1 \leq n$;

(b) $X = S^m$ and $Y = S^n$ for $0 \leq m < n$;

(c) $X = S^n$ and $Y = T^n$ for $n \geq 2$;

(d) $X = \mathbb{CP}^n$ and $Y = S^{2n}$ for $n \geq 1$;

(e) $X = \mathbb{CP}^n$ and $Y = S^1$ for $n \geq 1$.

Aufgabe 8. Let (X,A) be a topological pair such that A is $(n-1)$-connected and X is n-connected for $n \in \{1,2,\ldots\} \amalg \{\infty\}$. Prove or disprove that (X,A) is n-connected.

[0]Hand-in Monday 21.10.